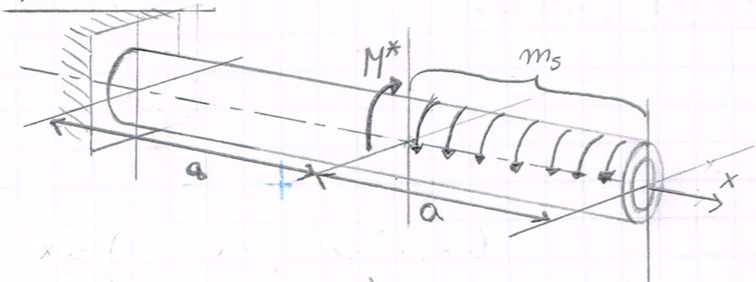
WYTRZYMAŁOŚĆ KONSTRUKCJI I   Maciej Jasiński

Seria VI   I = 6   N = 8

Zadanie 1

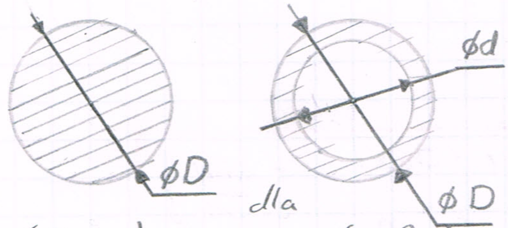


$a = 0{,}5\ m$

$D = 5{,}16\ cm \qquad d = 3{,}62\ cm$

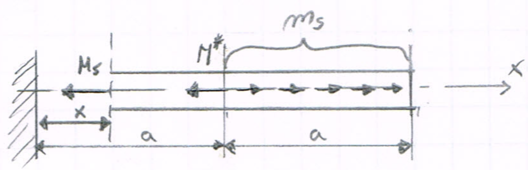
$M^* = 5{,}16\ kNm \qquad m_s = 5{,}12\ \frac{kNm}{m}$

$E = 2\cdot 10^5\ MPa \qquad \nu = 0{,}3$



dla $x \in (0, a)$ dla $x \in (a, 2a)$

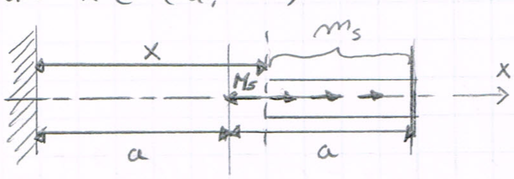
dla $x \in (0, a)$



$$-M_s(x) - M^* + \int_a^{2a} m_s\, dx = 0$$

$$M_s(x) = m_s \cdot a - M^* = -2{,}6\ kNm$$

dla $x \in (a, 2a)$



$$-M_s(x) + \int_x^{2a} m_s\, dx = 0$$

$$M_s(x) = 2 m_s a - m_s x = 5{,}12 - 5{,}12x\ [kNm]$$

$$\boxed{M_s(x) = \begin{cases} -2{,}6\ kNm, & x \in (0, a) \\ 5{,}12 - 5{,}12x\ [kNm], & x \in (a, 2a) \end{cases}}$$

dla $x \in (0, a)$

$$J_o = \frac{\pi}{32} D^4 \approx 6{,}960 \cdot 10^{-7}\ m^4$$

$$\tau_{max}(x) = \frac{M_s(x) D}{2 \cdot J_o} = \frac{M_s(x)\cancel{D}^{\,3}}{\cancel{2}\cdot \frac{\pi}{\cancel{32}_{16}} D^{4}} \approx -96{,}38\ MPa$$

dla $x \in (a, 2a)$

$$J_o = \frac{\pi}{32}(D^4 - d^4) = 5{,}274 \cdot 10^{-7}\ m^4$$

$$\tau_{max}(x) = \frac{M_s(x)\cdot D}{2\, J_o} \approx 250{,}5 - 250{,}5 \cdot x\ [MPa]$$

$$\tau_{max}(x) = \begin{cases} -96{,}38 & , \text{dla } x \in (0,a) \\ 250{,}5 - 250{,}5 \cdot x & , \text{dla } x \in (a, 2a) \end{cases} \quad [MPa]$$

moduł Kirchhoffa: $G = \frac{E}{2(1+\nu)} \approx 7{,}692 \cdot 10^4 \ MPa$

dla $x \in (0, a)$

$$\Theta(x) = \frac{2 \cdot \tau_{max}(x)}{G \cdot D} \approx -0{,}04857 \ \frac{rad}{m}$$

dla $x \in (a, 2a)$

$$\Theta(x) = \frac{2 \cdot \tau_{max}(x)}{G \cdot D} \approx 0{,}1262(1-x) \ \frac{rad}{m}$$

$$\Theta(x) = \begin{cases} -0{,}04857 & , \text{dla } x \in (0,a) \\ 0{,}1262(1-x) & , \text{dla } x \in (a, 2a \end{cases} \quad \left[\frac{rad}{m}\right]$$

$$\varphi(x) = \int_0^x \Theta(x)\,dx$$

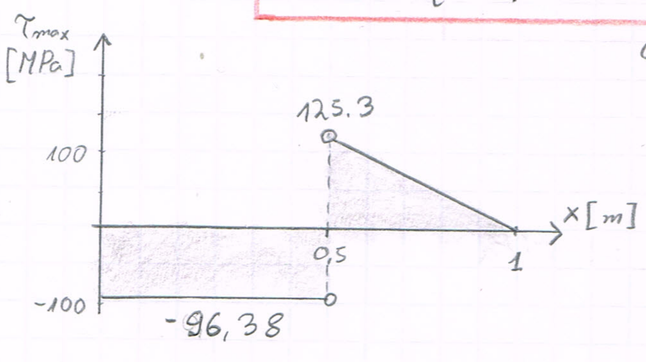
dla $x \in (0, a)$

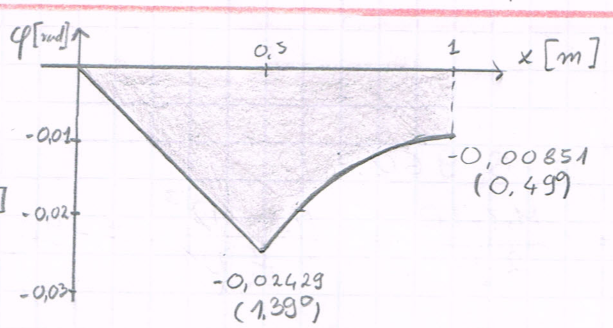
$$\varphi(x) = \int_0^x \Theta(x)\,dx = -0{,}04857x \ \ rad$$

dla $x \in (a, 2a)$

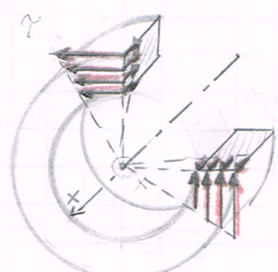
$$\varphi(x) = \int_0^a \Theta(x)\,dx + \int_a^x \Theta(x)\,dx = -0{,}07161 + 0{,}1262 \cdot x - 0{,}06310 \cdot x^2 \ [rad]$$

$$\varphi(x) = \begin{cases} -0{,}04857 \cdot x & , \text{dla } x \in (0,a) \\ -0{,}07161 + 0{,}1262 \cdot x - 0{,}06310 \cdot x^2 & , \text{dla } x \in (a, 2a) \end{cases} \quad [rad]$$





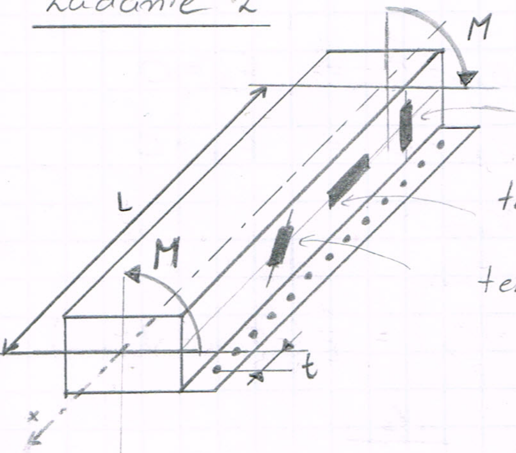
Przekrój najbardziej wytężony występuje dla $x \to a^+$

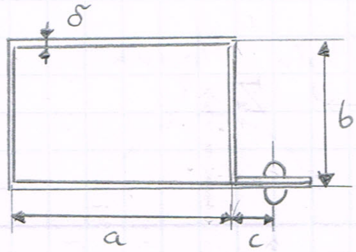


$\tau_{max} = \tau_D = 125{,}3M$

$\tau_d = \tau_{max}\frac{d}{D} \approx 87{,}9M$

Zadanie 2





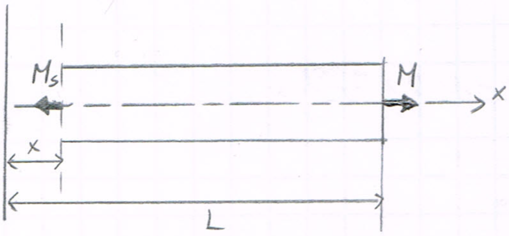
$M = 1{,}12 \ kNm$

$G = 2{,}6 \cdot 10^4 \ MPa$

$R_{0,2} = 280 \ MPa$

$a = 158mm \quad b = 56mm \quad c = 58mm$

$t = 25mm \quad L = 1m \quad \delta = 1mm$



$-M_s(x) + M = 0 \Rightarrow M_s(x) = M = 1{,}12 \ kNm$

Pole:

$A = a \cdot b = 8{,}848 \cdot 10^{-3} m^2$

I wzór Bredta:

$$\tau_{max} = \frac{M_s}{2A\delta}$$

$$\sigma_{red} \leq k_r = \frac{R_{0,2}}{n_e} \rightarrow n_e \geq \frac{R_{0,2}}{\sigma_{red}}$$

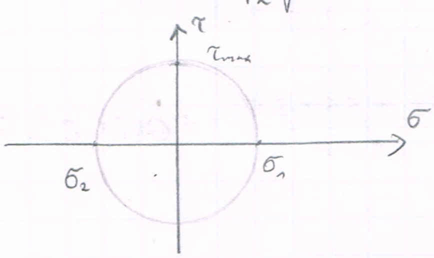
Wg. hipotezy Treski:

$$\sigma_{red}^T = 2\tau_{max} = \frac{M_s}{A\delta}$$

$$n_e^T \geq \frac{R_{0,2} \cdot A\delta}{M_s} \approx \frac{280 \cdot 10^6 Pa \cdot 8{,}848 \cdot 10^{-3} m^2 \cdot 10^{-3} m}{1{,}12 \cdot 10^3 \ Nm} \approx 2{,}212$$

wg hipotezy HMH:

$$\sigma_{red}^{HMH} = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} = \frac{1}{\sqrt{2}}\sqrt{4\tau_{max}^2 + \tau_{max}^2 + \tau_{max}^2}$$



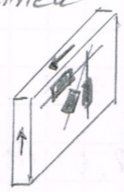
$\Rightarrow \quad \sigma_1 = \tau_{max} \quad \sigma_2 = -\tau_{max} \quad \sigma_3 = 0$

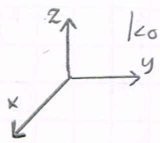
$$\sigma_{red}^{HMH} = \sqrt{3}\,\tau_{max} = \frac{\sqrt{3}}{2}\frac{M_s}{A\delta}$$

$$n_e^{HMH} \geq \frac{2}{\sqrt{3}}\frac{R_{0,2} A\delta}{M_s} \approx \frac{2}{\sqrt{3}} \frac{280 \cdot 10^6 Pa \cdot 8{,}848 \cdot 10^{-3} m^2 \cdot 10^{-3} m}{1{,}12 \cdot 10^3 \ Nm} \approx 2{,}554$$

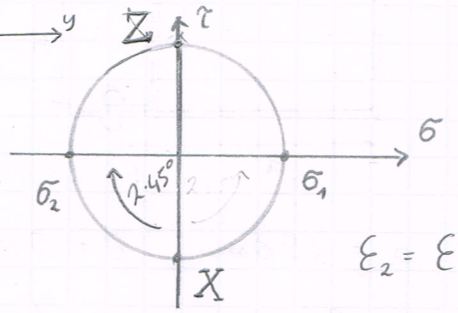
$$\boxed{\begin{array}{l} n_e^T \geq 2{,}212 \\ n_e^{HMH} \geq 2{,}554 \end{array}}$$

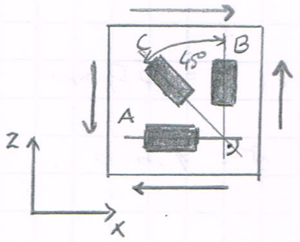
wycinek ścianki z prawej strony:





koło Mohra:





$\sigma_x = \sigma_A = 0 \Rightarrow \varepsilon_A = 0$

$\sigma_z = \sigma_B = 0 \Rightarrow \varepsilon_B = 0$

$\sigma_1 = \tau_{max}$

$\sigma_2 = -\tau_{max}$

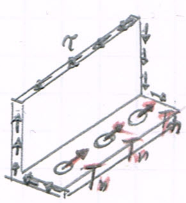
$$\varepsilon_2 = \varepsilon_c = \frac{1}{E}(\sigma_2 - \nu\sigma_1) = \frac{1}{E}(-\tau_{max} - \nu\tau_{max})$$

$$\varepsilon_c = -\frac{1}{2}\tau_{max}\frac{2(1+\nu)}{E} = -\frac{\tau_{max}}{2G} = -\frac{M_s}{4GA\delta}$$

$$\varepsilon_c \approx -\frac{1,12\cdot 10^3\,Nm}{4\cdot 2,6\cdot 10^{10}\,Pa\cdot 8,848\cdot 10^{-3}\,m^2\cdot 10^{-3}\,m} \approx -1,217‰$$

$$\boxed{\varepsilon_A = 0 \qquad \varepsilon_c \approx -1,217‰ \qquad \varepsilon_B = 0}$$

Siła w nicie:



$$\tau\cdot l\cdot\delta - T_n\cdot n = 0 \qquad \frac{l}{n} = t$$

$$T_n = \tau\cdot t\cdot\delta = \frac{M_s}{2A\cancel{\delta}}\cdot t\cancel{\delta} = \frac{M_s}{2A}t$$

$$T_n \approx \frac{1,12\cdot 10^3\,Nm}{2\cdot 8,848\cdot 10^{-3}\,m^2}\cdot 25\cdot 10^{-3}\,m \approx 1,582\,kN$$

$$\boxed{T_n = 1,582\,kN}$$

Całkowity kąt skręcenia:

II wzór Bredta:
$$\Theta = \frac{M_s}{4\cdot GA^2}\oint\frac{ds}{\delta} = \frac{M_s}{\underset{2}{\cancel{4}}GA^2}\cdot\frac{2a+2b+2c}{\delta}$$

$$\Theta \approx \frac{1,12\cdot 10^3\,Nm}{2\cdot 2,6\cdot 10^{10}\,Pa\cdot(8,848\cdot 10^{-3}\,m^2)^2}\cdot\frac{158\,mm + 56\,mm + 58\,mm}{1\,mm} \approx 0,07483\,\frac{rad}{m}$$

$$\varphi(x) = \int_0^x \Theta\,dx = \Theta\cdot x$$

$$\boxed{\varphi(l) = 0,07483\,rad \approx 4,29^\circ}$$